Spring Force Constant Determination as a Learning Tool for Graphing and Modeling

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One of the goals of science is the development of physical and mathematical models to describe physical systems by using observational and experimental data. We then use these models to either explain previously observed data or to predict results that have not actually been observed where the quality of the model determines its predictive value. In this experiment, we focused on developing a mathematical model relating the applied force on a spring and the resulting change in length (or stretching). We suspended weights of known masses (ranging from 0 g up to 270 g) from a randomly chosen spring and measured the changes in length of the spring. We then plotted the change in length (m) against the force (N) exerted by the mass on the spring for all our data points. From our data, we saw a clear linear relationship between force and displacement. Using linear regression, we determined our spring force constant, F_s, to be 21.3 N/m and the initial tension, T_{init}, of our spring to be approximately 0.5 N. Our results correlated nicely with Hooke's Law, which provides a general mathematical model for springs under compression and extension, but we further refined the prevailing mathematical model by incorporating T_{init}.

I. INTRODUCTION

This lab focuses on generating a model relating the force applied to a spring and the distance the spring stretches from its original length, or *rest length*. This relationship is well understood as Hooke's Law and states 1) extension of a spring is proportional to the applied force and 2) a spring will return to its rest length when the force is removed so long as the *elastic limit* has not been exceeded. Beyond the *elastic limit*, springs exhibit *plastic behavior* where additional force causes deformation of the spring such that the original or rest length is altered. Hooke's Law is illustrated in Fig. 1[1].

Mathematically, Hooke's Law can be described in Eqn. 1[2] where F_{s_w} is equal in magnitude to both N_{w_s} and W_{e_w} (the applied force), k is the spring's force constant, which is unique for any given spring and is a measure of the spring's stiffness, and d is the displacement change in length of the spring from its rest position.

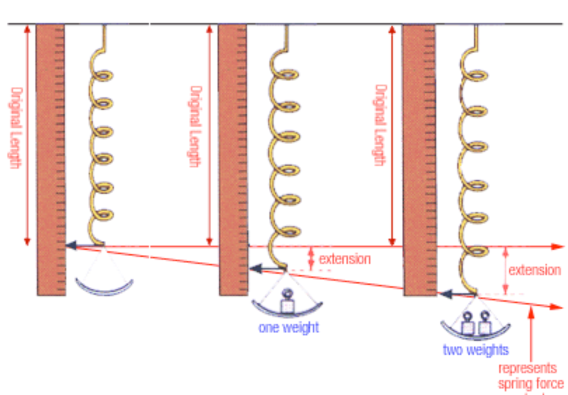

Fig. 1. Illustration of Hooke's Law. As additional weights are added, there is a linear increase in the length of the spring.

$$\left|F_{s\square w}\right| = \left|W_{e\square w}\right| = \left(k * d\right) = \left(m_{weights} * g\right) \quad \textbf{(1)}$$

The free body diagrams describing the elements in this system are in seen Fig. 2. Notice that F_{s_w} and N_{w_s} are Newton 3rd law pairs.

We are specifically interested in experimentally determining the spring constant, k, for our given spring. This spring constant arises from various physical

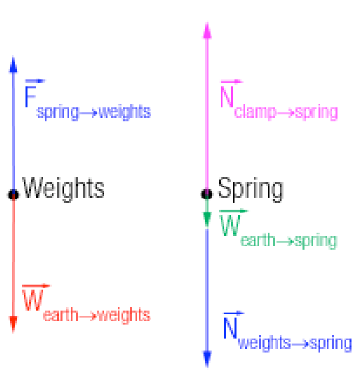

Fig. 2. Simple free body diagrams illustrating relationship between forces present in our model.

properties of the spring including the material it is made from, the number of coils, the diameter of the coils, etc.

This report will describe how we performed our measurements and calculated our data points. We will then analyze our data and create a mathematical model to describe our spring's behavior while experiencing a load. From the perspective of this lab as a learning tool, we will expand on the basic lab techniques that we practiced, how we calculated our values and what we learned while performing this lab.

We hypothesized that as we increased the force $N_{weights\_spring}$, the spring would lengthen. Since we were aware of Hooke's Law before starting, we also anticipated a linear relationship between this applied force and the change in length.

When analyzing our data we have to take into account the initial tension, T_{init}, in our spring. To ensure consistent rest lengths, most spring manufacturers design extension springs with an initial tension, which keeps the coils pressed tightly together. Hooke's Law may not work for small applied forces, as you must first overcome any initial tension before you see any apparent change in length[3].

II. MATERIALS AND METHODS

<u>Equipment Used</u>

- Spring (extension)
- Table clamp
- Weight hanger
- Weights (of known masses)
- String (of negligible weight)
- Lab scale

We first chose a random spring from the pool of available springs. We measured the length of this spring as well as the length and mass of the weight hanger. We then set up the table clamp, spring and weight hanger as illustrated in Fig. 3. We also set up a meter stick next to this apparatus so we could easily and consistently make length measurements. To reduce variability in measurements, we had only one group member make all measurements while other group members focused on tracking data and organizing our efforts.

We then proceeded to add weights to the weight hanger and record the change in length, d (m), of the spring (see Eqn. 2) where L is the measured length on the meter stick.

$$d = L_{total} - \left(L_{spring\_rest} + L_{weight\_hanger}\right) \quad \textbf{(2)}$$

We calculated the total force, F (N), at each data point

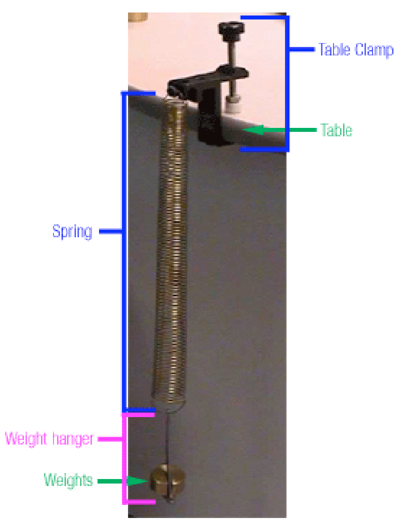

Fig. 3. Apparatus and spring setup using mass hanger for weights ≥ 50 g (0.5 N). Note, this was not our spring, but merely illustrates the setup.

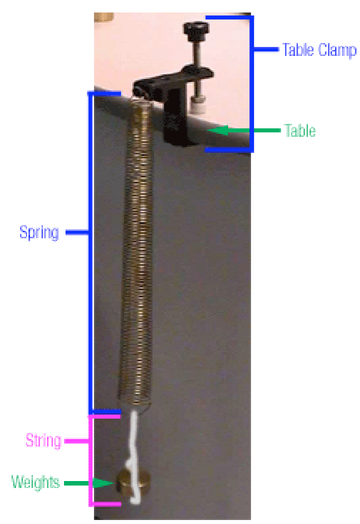

Fig. 4. Apparatus and spring setup using string to hold weights < 50 g (0.5 N) to the spring. Note, this was not our spring, but merely illustrates the setup.

using Newton's Second Law (Eqn. 3) and taking into account the total mass applied to the spring (Eqn. 4) where ***g*** is the force of gravity (9.80 m/s^2).

$$\vec{F} = m\vec{a} \quad \textbf{(3)}$$

$$F = \left(m_{\text{weights}} + m_{\text{weight\_hanger}}\right) * g \quad \textbf{(4)}$$

We were also interested in gathering some data points for masses less than 50 g. Since the weight hanger had an inherent mass of 50 g, to make smaller measurements, we had to remove the weight hanger and use a piece of string (negligible weight) to tie small weights to the spring (see Fig. 4). To calculate forces for these data points, we used Eqn. 5.

$$F = m_{\text{weights}} * g \quad \textbf{(5)}$$

When measuring the change in length, our uncertainty was ± 1 mm. Since the weights were of known calibrated masses, we did not include any uncertainty for them. However, we did measure the mass of the weight hanger and have to include the uncertainly in its measurement as ± 0.1 g.

As we made our measurements, we entered these into Microsoft Excel 1997 for analysis. We graphed changes in spring length versus applied force, added linear trend lines and used these to calculate the spring force constant.

III. RESULTS

Our group chose a small spring (approximately 1 cm

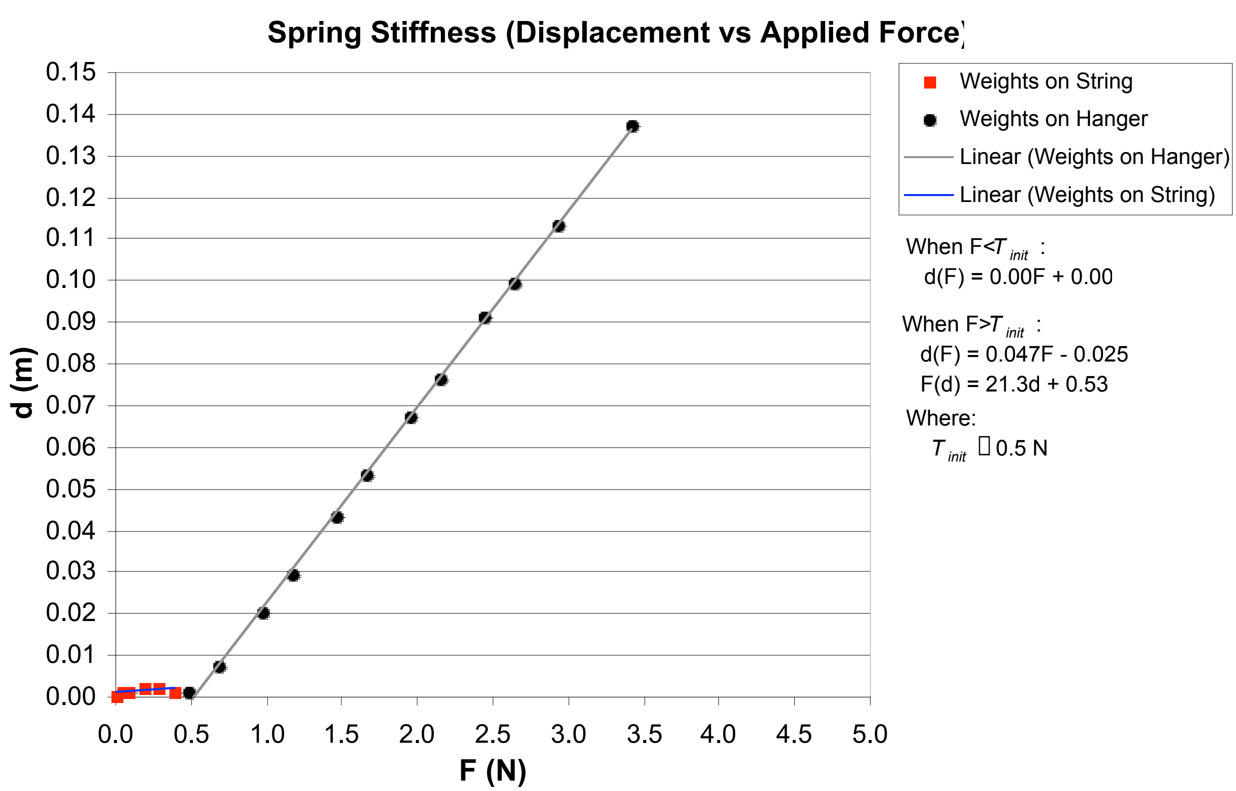

Fig. 5. Relates change (or displacement) in spring length, ***d*** (m), as a function of applied force (or load), ***F*** (N), on the spring. The "Weights on String" data series (indicated by points with red squares) were measure by suspending the weights using a string of negligible weight. These points show no change in ***d*** per unit increase in ***F*** and illustrate that the applied force ***F*** is less than the initial tension T_{init} (~ 0.53 N) of the spring. The data "Weights on Hanger" (indicated by black circles) indicate data points taken using a "mass hanger" rather than a string to hold the weights. The mass of this hanger (50 g ± 0.1 g) was taken into account when computing the ***F*** values. We see from the trend lines that once ***F*** exceeds T_{init}, we have a clear linear relationship between ***d*** and ***F***. The calculated slope of loads greater than 0.5 N is 0.047 m/N, which gives us a spring force constant of ***k*** = 21.3 N/m for this spring. Note that the uncertainty in ***d*** is ± 0.001 m and these error bars are present, but are not easily seen as they are very close to the plotted points.

diameter, 8 cm long and 10 g) for our experiment. Our observed measurements are graphed in Fig. 5. For weights less than 0.5 N, we saw no appreciable change in displacement and the spring coils remained tightly together (r = 0.641, se = 0.000). Over this range, the graph is a horizontal line following the x-axis. For weights greater than 0.5 N, we saw a very strong linear correlation (r = 1.000, se = 0.001) between increased weight, $\boldsymbol{F}$ (N), and increased stretching, $\boldsymbol{d}$ (m), of our spring. We took measurements up to 3.5 N but did not go any higher for fear of exceeding the *elastic limit* of our spring and entering the *plastic zone* (at which point the spring starts permanently deforming). In Fig. 5, we separated our data points based on how we attached the weights to the spring. For weights less than 0.5 N, we used a piece of string (these points are indicated with red squares). For weights 0.5 N and greater, we used a standard mass hanger (of weight 0.5 N) and additional weights to total the desired test amount. MS Excel utilized linear regression to generate the trend lines. We calculated our spring force constant, $\boldsymbol{k}$, to be 21.3 N/m (using Eqn. 1) and we observed an initial tension, $\boldsymbol{T_{init}}$, in our spring of approximately 0.5 N.

IV. DISCUSSION

From our data, our spring clearly obeyed Hooke's law as we anticipated. Given the size of our spring, the spring force constant of 21.3 N/m seemed quite reasonable. However, two surprising results were 1) how closely our data fit a linear model, and 2) how dramatic the initial tension appeared in our graph.

Typically, when scientists gather data and attempt to create mathematical models, they find general correlations and trends, but usually see variation in observed data versus predicted values from the model. Most often, the model will describe the general trend but individual data points will have a noticeable departure from the predicted values. Using statistical measures, we found a correlation coefficient of r = 1.000 and a standard error of se = 0.001. This indicated that 1) there was an absolute correlation between an increase in applied force and an increase in spring length, and 2) the data points we observed were almost a perfect fit to our linear model.

The second surprise was the dramatic shift in our trend lines at about 0.5 N (see Fig. 5). We had anticipated that our spring would obey Hooke's Law, but before beginning the experiment, we had not heard of the property of *initial tension* in a spring. While interpreting our data, we researched this phenomenon and found that most extension springs (as opposed to compression springs) are manufactured with the coils tightly pressed against each other. This ensures consistent *rest length* (a feature that many customers look for). However, this initial tension must first be overcome before a spring will stretch. From our graph, the initial tension in our spring was approximately 0.5 N. Our graph does not take into account the weight of the spring itself as it was suspended. To obtain a more accurate measure of $\boldsymbol{T_{init}}$, we would need to repeat the 0 – 0.5 N data points in a horizontal rather that vertical format with the spring resting on a near frictionless surface. In this manner, the weight of the spring would neither increase nor decrease the apparent $\boldsymbol{T_{init}}$. Because our spring's weight was low relative to the 0.5 N observed tension, we can say that the initial tension is close to 0.5 N, but in reality should be slightly greater (subtracting the weight of the spring).

Looking back at our initial model, we should probably modify it to take into account $\boldsymbol{T_{init}}$. Revisiting Eqn. 1 for our spring yields:

$$F_{s \to w} = W_{e \to w} = (T_{init} + k * d) \qquad \textbf{(6)}$$

$$d = \frac{W_{e \to w} - T_{init}}{k} \qquad \textbf{(7)}$$

$$d = \frac{W_{e \to w} - 0.5N}{21.3\,{}^{N}\!/_{m}} \qquad \textbf{(8)}$$

Unlike the original proposed Eqn. 1 (from Hooke's Law), Eqn. 6 and Eqn. 7 should accurately model extension springs (whether or not they have any initial tension) and Eqn. 8 specifically models our spring.

V. CONCLUSION

During this lab, we gained practice in making basic physical measurements, performing simple calculations and analyzing data sets to create a model for a physical system (the stretching of a spring). By taking data on our system we clearly developed a model stating, "For forces greater than 0.5 N, our spring will stretch 0.047 m per 1.0 N of applied force." Half of this statement was quite expected and fits with known and well-established properties of springs. However, the other half of our model (*i.e.* the "discovery" of *initial tension* in our spring of ~ 0.5 N) illustrates the importance of understanding what the data means and realizing that models must fit with observations not the other way around. Where discrepancies exist, it is the model that is usually found to be lacking. In fact, the unusual and unexpected data often prove to be far more interesting than the expected data. This we believe is the take home message from the experiment.

ACKNOWLEDGEMENTS

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[1] Used with permission. Original artwork from *Physics for You*: Nelson Thornes Publishers, adapted by Andy Darvill with further adaptations by Alfred Einstein. www.darvill.clara.net/enforcemot/springs.htm

[2] Serway, Beichner. 2000. *Physics for Scientist and Engineers Volume 1, 5th ed.*, pg 192. Orlando, Florida: Saunders College Publishing.

[3] General Wire Spring Company website. www.generalwirespring.com/extension.html